

Name:
Score:

Math 441
Final Exam

Read all directions and answer the questions the best that you can. Reasonable work is expected to be shown for all problems. The test contains 164 points and will be graded out of 160 (hence there are 4 bonus points built in).

Part 1 (4 points each)

Use the graphs of $f(x)$ and $g(x)$ to answer the following problems.

1.) $\lim_{x \rightarrow -2} f(x) =$

2.) $\lim_{x \rightarrow 1} f(x) =$

3.) $\lim_{x \rightarrow \infty} f(x) =$

4.) Where is $f(x)$ continuous?

5.) Where does $g(x)$ have critical numbers?

6.) Where is $g(x)$ concave up?

Part 2 (5 points each)
Evaluate the following limits.

1.) $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x^2 - 2x}$

2.) $\lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$

3.) $\lim_{x \rightarrow \infty} \frac{x^3 - 4x + 20}{9 - 2x^3}$

4.) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{x}$

$$5.) \lim_{x \rightarrow 1} \frac{\frac{1}{x+1} - \frac{1}{2}}{x - 1}$$

$$6.) \lim_{x \rightarrow 2^-} \frac{2x - 6}{x - 2}$$

$$7.) \lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$$

Part 3 (5 points each)

Find the derivative for the following functions. You do not have to simplify your answer.

$$1.) a(x) = (x^3 - 1)(2x + 4)$$

2.) $b(x) = \tan x - 3 \csc x$

3.) $c(x) = \sec(x^4) + \sqrt{2x^3 - 4}$

4.) $d(x) = \frac{x^3 - 2x + 4}{x^2 + x}$

5.) $e(x) = \sqrt[4]{x^3} - \frac{5}{x^3}$

Part 4 (5 points each)

Evaluate the following integrals. Answers do not have to be simplified.

1.) $\int x^4 - 6x^2 + 5x - 2 \, dx$

2.) $\int \csc^2 \theta - 5 \cos \theta \, d\theta$

3.) $\int_1^3 \frac{x^5 - 2x^3}{x^3} \, dx$

4.) $\int 12x^2 \sin(x^3 - 2) \, dx$

Part 5 (6 points each)

Answer the following 6 questions. Support your answer using the tools of Calculus.

1.) Where is $z(x) = \frac{3x+6}{x^4-9x^2}$ continuous?

2.) What is the equation of the line tangent to $y(x) = x^4 - 2x^3 + 6x - 3$ at $x = -1$?

3.) Find the critical numbers of the function $p(x) = \frac{1}{3}x^3 - 5x^2 + 9x - 1$ and classify each as a local max, min or neither.

4.) Where is $m(x) = \frac{x^2 + 12}{x + 2}$ increasing and decreasing?

5.) Where is the function $l(x) = \frac{1}{20}x^5 + \frac{1}{4}x^4 - 2x + 10$ concave up? Concave down?

6.) Find the exact area under $j(x) = x^3 - 5x + 4$ from $x = -1$ to $x = 1$.

Part 6 (8 points each)

Answer each of the following problems.

1.) Given an object moves according to the function $s(t) = (t^3 - 2t)^2$, find functions that measure the object's velocity and acceleration.

2.) You have a square piece of cardboard, each side is 4 feet long. You cut equal squares off each corner of the square, fold up the resultant sides to form an open box. How long should each cut be in order to make a box of largest possible volume?

3.) Each side of a square is increasing at a constant rate of 8 centimeters per second. At what rate is the area of the square increasing when the area of the square is 16 square centimeters? Include units with your answer.